

CHAPMAN & HALL/CRC APPLIED MATHEMATICS
AND NONLINEAR SCIENCE SERIES

Multi-Resolution Methods for Modeling and Control of Dynamical Systems

Puneet Singla
John L. Junkins



CRC Press

Taylor & Francis Group
Boca Raton London New York

CRC Press is an imprint of the
Taylor & Francis Group, an **Informa** business
A CHAPMAN & HALL BOOK

Contents

1	Least Squares Methods	1
1.1	Introduction	1
1.2	The Least Squares Algorithm	2
1.3	Linear Least Squares Methods	3
1.3.1	Batch Least Squares Method	3
1.3.2	Sequential Least Squares Algorithm	5
1.4	Non-Linear Least Squares Algorithm	8
1.5	Properties of Least Squares Algorithms	10
1.6	Examples	11
1.6.1	Smooth Function Approximation	11
1.6.2	Star Camera Calibration	12
1.7	Summary	19
2	Polynomial Approximation	21
2.1	Introduction	21
2.2	Gram-Schmidt Procedure of Orthogonalization	22
2.2.1	Three-Term Recurrence Relation to Generate Orthogonal Polynomials	24
2.2.2	Uniqueness of Orthogonal Polynomials	25
2.3	Hypergeometric Function Approach to Generate Orthogonal Polynomials	30
2.3.1	Derivation of Rodrigues's Formula for Continuous Variable Polynomials	34
2.3.2	Leading Coefficients for Three-Term Recurrence Formula	36
2.4	Discrete Variable Orthogonal Polynomials	38
2.4.1	Hypergeometric Type Difference Equation	39
2.4.2	Derivation of Rodrigues's Formula for Discrete Variable Orthogonal Polynomials	42
2.4.3	Leading Coefficients for Three-Term Recurrence Formula for Discrete Variable Orthogonal Polynomials	44
2.5	Approximation Properties of Orthogonal Polynomials	45
2.6	Summary	48

3	Artificial Neural Networks for Input-Output Approximation	49
3.1	Introduction	49
3.1.1	Radial Basis Function Networks	50
3.2	Direction-Dependent Approach	55
3.3	Directed Connectivity Graph	60
3.3.1	Estimation Algorithm	62
3.3.2	Spectral Decomposition of the Covariance Matrix	64
3.3.3	Additive Decomposition of the Covariance Matrix	66
3.3.4	Cholesky Decomposition of the Covariance Matrix	66
3.4	Modified Minimal Resource Allocating Algorithm (MMRAN)	69
3.5	Numerical Simulation Examples	72
3.5.1	Test Example 1: Function Approximation	73
3.5.2	Test Example 2: 3-Input 1-Output Continuous Function Approximation	80
3.5.3	Test Example 3: Dynamical System Identification	82
3.5.4	Test Example 4: Chaotic Time Series Prediction	86
3.5.5	Test Example 5: Benchmark Against the On-Line Structural Adaptive Hybrid Learning (ONSAHL) Algorithm	90
3.6	Summary	93
4	Multi-Resolution Approximation Methods	95
4.1	Introduction	95
4.2	Wavelets	97
4.3	Bèzier Spline	105
4.4	Moving Least Squares Method	110
4.5	Adaptive Multi-Resolution Algorithm	112
4.6	Numerical Results	116
4.6.1	Calibration of Vision Sensors	116
4.6.2	Simulation and Results	117
4.6.3	DCG Approximation Result	119
4.6.4	Local Approximation Results	121
4.7	Summary	121
5	Global Local Orthogonal Polynomial MAPping (GLO-MAP) in N Dimensions	123
5.1	Introduction	123
5.2	Basic Ideas	125
5.3	Approximation in 1, 2 and N Dimensions Using Weighting Functions	128
5.4	Global-Local Orthogonal Approximation in 1-, 2- and N -Dimensional Spaces	136
5.4.1	1-Dimensional Case	139
5.4.2	2-Dimensional Case	140

5.4.3	<i>N</i> -Dimensional Case	142
5.5	Algorithm Implementation	144
5.5.1	Sequential Version of the GLO-MAP Algorithm	146
5.6	Properties of GLO-MAP Approximation	149
5.6.1	Approximation Error	149
5.6.2	Bounds on Approximation Error	150
5.6.3	Probabilistic Analysis of the GLO-MAP Algorithm . .	152
5.7	Illustrative Engineering Applications	155
5.7.1	Function Approximation	155
5.7.2	Synthetic Jet Actuator Modeling	160
5.7.3	Space-Based Radar (SBR) Antenna Shape Approximation	166
5.7.4	Porkchop Plot Approximations for Mission to Near-Earth Objects (NEOs)	170
5.8	Summary	174
6	Nonlinear System Identification	179
6.1	Introduction	179
6.2	Problem Statement and Background	180
6.3	Novel System Identification Algorithm	182
6.3.1	Linear System Identification	185
6.3.2	State Variable Estimation	189
6.4	Nonlinear System Identification Algorithm	190
6.4.1	Learning Algorithm for State Model Perturbation Approach (SysID 1)	190
6.4.2	Learning Algorithm for Output Model Perturbation Approach (SysID 2)	197
6.5	Numerical Simulation	199
6.5.1	Dynamic System Identification of Large Space Antenna	199
6.6	Summary	205
7	Distributed Parameter Systems	207
7.1	Introduction	207
7.2	MLPG-Moving Least Squares Approach	210
7.2.1	Poisson Equation	214
7.2.2	Comments on the MLPG Algorithm	220
7.3	Partition of Unity Finite Element Method	222
7.3.1	Poisson Equation	228
7.3.2	Fokker-Planck-Kolmogorov Equation	235
7.4	Summary	240

8	Control Distribution for Over-Actuated Systems	243
8.1	Introduction	243
8.2	Problem Statement and Background	245
8.3	Control Distribution Functions	251
8.3.1	Radial Basis Functions	254
8.3.2	Global Local Orthogonal Basis Functions	255
8.4	Hierarchical Control Distribution Algorithm	259
8.5	Numerical Results	264
8.5.1	Control Allocation for a Morphing Wing	264
8.6	Summary	273
	Appendix	275
	References	285
	Index	297